Counting Hexagons

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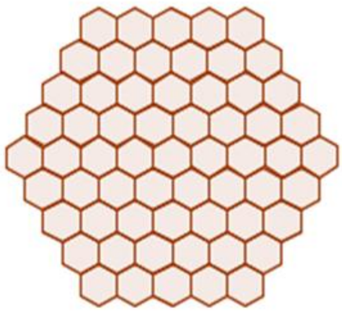
Problem

When making a cable for a suspension bridge, many strands are assembled into a hexagonal formation and then compacted together. The diagram below illustrates a 'size'5 cable made up of 61 strands.



How many strands are needed for a size 10 cable? How many for a size n cable?

The solutions are ordered according to how I thought of it. Before I thought of the visual solutions I have to know first what kind of function relates the number of cable on the sides and the total number of cables.

No. strands at the side	**1**	**2**	**3**	**4**	**5**
Total no. of strands	**1**	**7**	**19**	**37**	**61**

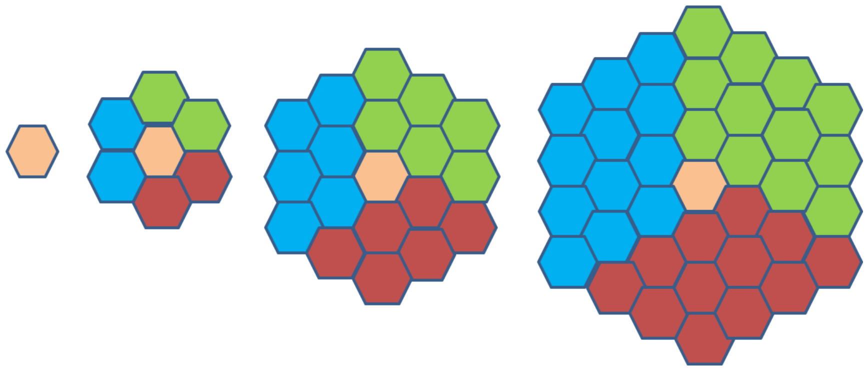
First difference 6 12 18 24

Second difference 6 6 6

Since the second difference is constant, the function must be quadratic. From here I can use any three ordered pairs to find the equation of the quadratic or some other technique. But I thought the visual solution is more interesting.

Knowing that the function is quadratic, all I need to do is to find a portion in the figure that will calculate an n by n array or any portion in the figure that will let me calculate n^2 or an $(n-1)^2$. Here are the solutions, according to the order of how I thought of it.

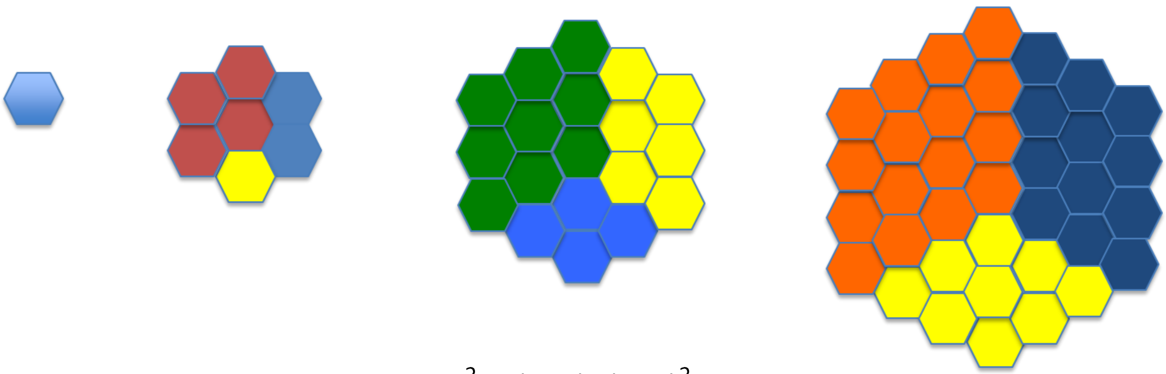
Solution 1



$3xn(n-1) +1$

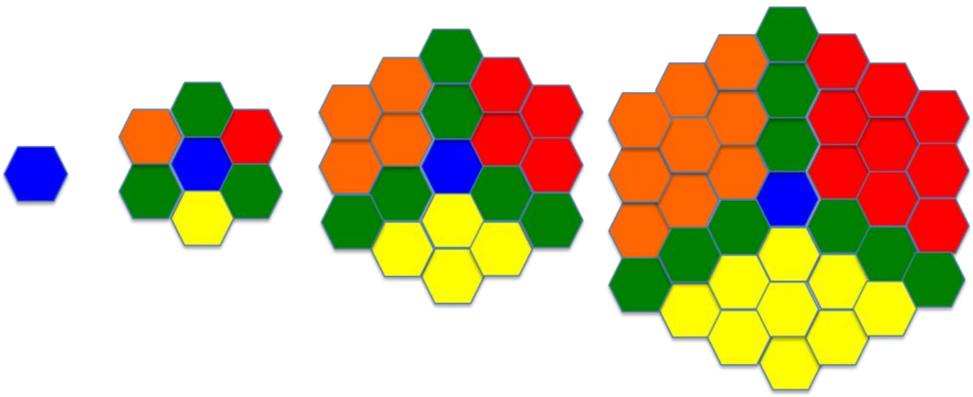
If n = 5, $5(5-1)x3+1 = 5x4x3+1 = 61$

Solution 2



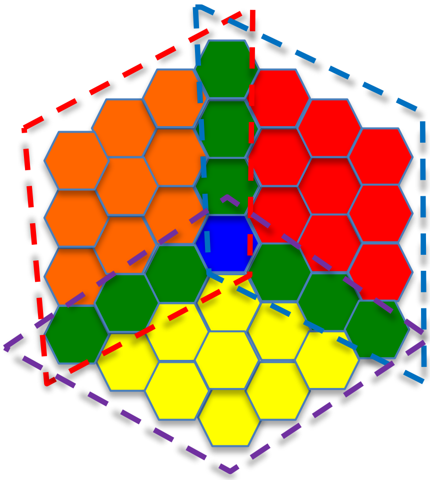
$n^2+n(n-1)+(n-1)^2$

Solution 3



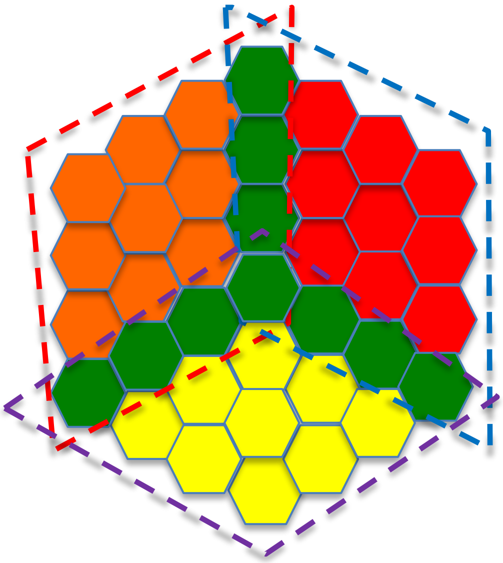
$3(n-1)^2 +3(n-1)+1$

Solution 4



$3n^2-3(n-1)-2$

Solution 5



$3n^2-3n+1$