Guide for Planning and Analyzing Mathematics Lessons in Lesson Study

Contents

Introduction and Purpose of the Guide

Introduction

Research supports the common-sense view that classroom instruction is a major influence on student achievement in mathematics. Although simplistic notions that student-centered instruction is superior to teacher-directed instruction, or vice-versa, are not supported by research (e.g. U.S. Dept of Education, 2008), research does support other conclusions about what is effective and what is not. For instance, in Everybody Counts, the National Research Council concluded that evidence from many sources shows that the least effective mode for mathematics learning is... lecturing and listening... For most students and most teachers mathematics continues to be primarily a passive activity: teachers prescribe; students transcribe. Students simply do not retain for long what they learn by imitation from lectures, worksheets, or routine homework. Presentation and repetition help students do well on standardized tests and lower-order skills, but they are generally ineffective as teaching strategies for long-term learning, for high-order thinking, and for versatile problem solving. (page 57)

On the positive side, Heibert and Grouws (2007) found that a focus on conceptual understanding—making the connections between and among mathematical ideas explicit—and expecting students to grapple with mathematical ideas instead of simply spoon-feeding them information, both contribute to significant improvements in student achievement.

The problem for teachers is how to design lessons with those features which maximize learning, a problem made more difficult by most teachers' lack of exposure to a variety of models of teaching. Most teachers have a fairly narrow, parochial view of what teaching should look like—a view based on their own experiences in school.

The Classroom Innovations through Lesson Study was created to help educators learn about Lesson Study so as to improve mathematics education. A major contribution of the site is the growing collection of Lesson Study Videos in mathematics. These videos provide a window into the different teaching practices in different countries, including countries which have historically been among the top performers in international comparisons of mathematics achievement.

These videotaped lessons are not put forward as models of perfect lessons, just as the goal of Lesson Study is not to produce a perfect lesson. The goal of Lesson Study, and the goal of this web site in making these videos available, is to maximize teacher learning. These videos provide the raw material for productive discussion for education who seek to understand what makes a lesson effective and to acquire new ideas for teaching.

The purpose and content of this guide

The purpose of this guide is to support the development of high-quality lessons and to help educators effectively analyze lessons, focusing on those elements which are associated with greater student learning.

The guide first outlines important elements to be considered when developing a lesson, including moving from telling to explaining to guiding; effectively situating a lesson within a unit; pacing the development of conceptual understanding; identifying key questions; and incorporating assessment. In addition to guiding effective planning, these elements also form a frame for the analysis and assessment of lesson quality.

Next the guide turns to more specific considerations for planning and implementing effective lessons, including clarity of objectives; alignment and quality of tasks, activities and problems; anticipating and responding to mistakes and misconceptions; and the skillful use of effective questioning. Once again, this set of considerations supports both lesson development as well as individual and collaboration lesson review and critique.

Finally to support a collegial process of thoughtful and constructive study of lessons, the guide provides detailed directions and questions for conducting analyses of lessons and guiding their revision and improvement. Examples of analyses of lessons from the Lesson Study Videos are provided to show how this guide might be used to frame discussion of a lesson.

The authors of this guide offer it as a checklist which might be used in the context of lesson study, for creating or analyzing lessons. It is hoped that, in the future, this guide may provide a focus for educators to discuss other lessons collaboratively by using wiki technology.

Principles for Lesson Planning

Implementing high quality lessons is hard, takes time, and requires deliberate planning. We are expected to find ways to make math work for far more kids. We live in a world that expects, even requires, deeper understanding and far greater problem-solving skill. That is why our lessons must be more carefully planned and why bare-bones lesson plans of objectives, examples, exercises, and homework need to be broadened.

Components of a thorough lesson plan

For every lesson, careful consideration should be given to the following:

- The mathematical content of the lesson. - What skills or concepts are being developed or mastered as a result of the lesson? Often, teachers who plan effective lessons back-map the content from asking, "Exactly what do I expect my students to know or be able to do at the end of this lesson?"
- The mathematical tasks of the lesson. - What specific questions, problems, tasks, investigations, or activities will students be working on during the lesson? Often, this includes the worksheets that are prepared for the lesson and identifies the references or materials that are needed.
- Evidence that the lesson was successful. - Deliberate consideration of what performances will convince you (and any outside observer) that most, if not all, of your students have accomplished your objective.
- Launch and closure. - How to use the first 5 minutes of the lesson, what connections to make to prior learning, and what summary will close the lesson.
- Notes and nuances. - Reminders about vocabulary, connections, common mistakes, and typical misconceptions that need to be considered before the lesson and kept in mind during it.
- Resources and homework. - What materials or resources are essential for students to successfully complete the lesson tasks or activities? What follow-up tasks will be assigned upon the completion of the lesson?
- Post-lesson reflections. - A home for the inevitable "If only …" realizations that should be noted to inform your planning the next time. (Leinwand, 2009)

Quality lessons have clear story lines

The curriculum analysis from the Third International Mathematics and Science Study (TIMSS – now called Trends in International Mathematics and Science Study) is that the curricula from the so-called A+ countries are much more focused and coherent (National Research Council, 1999). In the United States, the National Council of Teachers of Mathematics argues for such curriculum coherence in their document, Curriculum Focal Points (NCTM, 2006).

Coherence is a critical factor not only for curriculum but also for teaching. As teachers plan, implement, analyze, and revise a mathematics lesson, they need to consider if there is a coherent story line in the lesson. Just as with any good story, a lesson should have a good beginning, development, and conclusion.

The storyline should be coherent mathematically and pedagogically. The problem that is posed at the beginning of the lesson should be the sound starting point. Mathematical ideas should be developed as students solve the initial problem. The classroom teacher should carefully orchestrate the classroom discourse so that students can deepen their understanding through careful and critical reflection on their ideas. Follow-up problems should be closely tied to the main idea developed by students. The lesson should end with a time where students can reflect and make explicit what they learned.

Quality lessons attend to all five aspects of mathematical proficiency

The goal of mathematics instruction is to help students become proficient in mathematics. The National Research Council defines “mathematical proficiency” to be made up of the following intertwined strands:

- conceptual understanding – comprehension of mathematical concepts, operations, and relations
- procedural fluency – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- strategic competence – ability to formulate, represent, and solve mathematical problems
- adaptive reasoning – capacity for logical thought, reflection, explanation, and justification
- productive disposition – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy. (NRC, 2001, p. 5)

These strands are intertwined. As we plan, implement, analyze and revise mathematics lessons, it is usually easy to focus on the first two strands. It is perhaps necessary to focus on a specific understanding goal. However, it is also critical to remember that these five strands are, in reality, inseparable. As students are given the main mathematical task of the lesson, students must clearly understand the problem – or formulate their own learning task. Students must realize that they can solve the problem and persist when they encounter difficulty. Students must also be able to explain their ideas to their classmates and teachers. Thus, even though the specific learning goal of the lesson may be a particular concept or the development of a particular procedure, teachers must realize that how they interact with their students during the lesson will influence their students' development of strategic competence, adaptive reasoning, and productive disposition. Because paying attention to all these strands on your feet is challenging, it is important for teachers to think about these ideas as they plan their lessons.

Quality lessons support teacher growth from Level 1 to Level 3 mathematic

As Polya begins his famous book, *How to Solve It* (1945), “The purpose of mathematics class is to help students, but not too much and not too little.” This means that good teachers should leave the student a reasonable share of the work and not give knowledge and procedures by spoon-feeding.

Japanese mathematics educators characterize teacher expertise according to three levels (Sugiyama 2008):

- Level 1 teachers can tell students the important basic ideas of mathematics such as facts, concepts, and procedures (teaching by telling).
- Level 2 teachers can explain the meanings and reasons of the important basic ideas of mathematics in order for students to understand them (teaching by explaining).
- Level 3 teachers can provide students opportunities to understand these basic ideas, and support their learning so that the students become independent learners (teaching based on students' independent work).

Japanese mathematics educators argue that it is not acceptable for teachers to stay at Level 1. Teachers should be at least at Level 2 and strive to become Level 3. Because achieving Level 3 is not easy, teachers need to work collaboratively with other teachers through lesson study to plan, examine, and reflect on their instruction.

Quality lessons consider misconceptions

As we plan, implement, analyze and revise mathematics lessons, students’ understanding should be the main focus. But, what does this really mean? Clearly, we should know what students currently understand so that we can devise a potential learning path toward new understandings. We must make sure that students have the sufficient prerequisite understanding to benefit from the tasks we plan to give them. We must also plan how we can lead classroom discussion to raise the level of students’ understanding. But, in addition to these mathematical considerations, there are other factors we must consider.

One important consideration is children’s common errors and misconceptions. Children make errors and acquire misconceptions because they think! But because their mathematical understanding is incomplete, because their experience is limited, their thinking leads them to make erroneous conclusions.

For example, all primary teachers are familiar with errors like 25 + 37=512. Children make this error because they do not understand that you can have one and only one numeral in each place. When students use materials such as base-10 blocks, there is nothing logically wrong with using 5 longs—10 connected unit cubes—and 12 unit cubes (representing 50 + 12), so it is not surprising that students might write this quantity as "512." They do not see the invisible place value columns that are so clear to teachers. Other examples include the misconception “multiplication makes bigger and division makes smaller” (e.g., Greer, 1992) and the mistake of multiplying the adjacent sides to calculate the area of all parallelograms. These mistakes and misconceptions occur because they appear to be reasonable in light of the experiences students have had.

Because these errors and misconceptions are derived from students' experiences, it is not usually enough to correct them quickly and move on; rather, they must be dealt with head on in lessons.

Another important factor is the use of appropriate tools in lessons. Those tools may include manipulatives (such as pattern blocks and base-10 blocks), technologies (such as calculators and computers), and diagrams (such as tape diagrams and double number line). Some tools will be instrumental in helping students solve problems and develop new understanding. On the other hand, if the focus is only on getting answers, tools can become simply crutches instead of scaffolds. Therefore, teachers must think carefully about whether or not a tool should be used, which tool to used, and how it will be presented to students.

How a problem is presented to students is also an important consideration. This question goes beyond how the problem should be worded, which is itself an important consideration. Will the problem be accompanied a visual representation? Will there be demonstrations or numerical examples to illustrate the problem situation? Will students participate in the demonstration? How students use various modalities to make sense of and attempt to solve the given problem also influences what and how students will learn.

So, “focusing on students” really means focusing on the whole students – not just their mathematical understanding but also their ways of thinking and their perceptions.

Quality lessons intertwine assessment and instruction

As we plan, implement, analyze, and revise mathematics lessons focusing on students’ mathematical proficiency, assessment plays a critical role before, during, and after each lesson. National Council of Teachers of Mathematics recommends that assessment should be more than merely a test at the end of instruction to see how students perform under special conditions; rather, it should be an integral part of instruction that informs and guides teachers as they make instructional decisions. Assessment should not merely be done to students; rather, it should also be done for students, to guide and enhance their learning. (NCTM, 2000, p. 22)

As teachers plan a lesson, they must use their assessment of their students’ current understanding. When a learning task is posed in a lesson, teachers must assess how well students understand the task. As students engage in the learning task, teachers must assess how students are approaching the task, perhaps comparing to anticipated solutions. As the whole class discusses various ideas proposed by students, teachers must constantly assess how well students are understanding and what questions might provide the necessary scaffolding for students to raise the level of their understanding. As the lesson concludes, teachers must devise a way to assess students’ understanding, which, in turn, will inform what needs to take place in the next day’s lesson. As teachers analyze a lesson, they must consider carefully what students come to understand in the lesson and what factors may have contributed to students’ learning (or lack thereof). Lessons can only be revised effectively based on the analysis of assessment results.

As you can see, when assessment becomes an integral part of instruction, planning, implementing, analyzing, and revising mathematics lessons cannot be separated from planning, implementing, analyzing and revising assessment.

Key Considerations in Planning and Implementing Effective Lessons

In Section II, we discussed some principles that frame planning, implementing, analyzing and revising mathematics lessons. In this section, we will discuss more specific ideas we must consider as we plan and implement mathematics lesson.

Setting clear objectives that enhance students' learning

A clear and learning objective (or a set of objectives) is essential for effective mathematics teaching. By definition, learning means change in students' understanding. Therefore, the learning objective is about what students will understand that they did not understand before the lesson. It should be about specific mathematics, stated at a grain size small enough to be achieved within a single lesson. It should not be about "understanding multiplication," for instance; it should be about understanding something more specific about multiplication.

Often times, lesson objectives are stated in the form of what students will do. But such objectives may not make clear how students' understanding should change. For example, let us consider the following objective statement: "Students will be able to find the area of an L-shape using their prior knowledge of how to calculate the area of rectangles and squares." This statement is about a specific mathematics topic, the area of compound figures. The statement also makes connections to students' prior learning. What is not clear in this statement is what students will understand when they can calculate the area of L-shape using their prior knowledge. Something like the following would be more complete: "Students will understand that they can find the area of an unfamiliar shape by changing the shape into familiar ones." One might want to include also some specific, but generalizable strategies: "Students will understand that the area of a compound shape may be calculated by (1) sub-dividing the given shape into a collection of familiar shapes, or (2) cutting and re-arranging the given shape to change it into a familiar shape."

A clear learning objective sets the tone of lesson planning and implementation. The learning objective will guide your selection of the learning task and how you will assess students' understanding throughout the lesson. It will also guide you as you consider other ideas that are discussed below. Of course, as you will see below, consideration of other important ideas may suggest that you need to refine your learning objective, too.

Linking the lesson content to the curriculum, the unit, prior lessons, and future lessons

Mathematics is a body of well-connected ideas. The specific mathematics you are teaching, therefore, is connected to many different ideas. Each new mathematical idea must be built upon prior knowledge and will in turn lead to other mathematical ideas.

When planning to teach a new topic, you need to examine the broader mathematical story line and identify the particular role this specific topic plays in the story. Questions to ask are, "What have my students studied previously that is related to this topic?" and "What will my students study in the future that will build upon this topic?" These questions may help you understand why a particular topic may be positioned at this particular point in the curriculum.

For example, let us consider the topic of finding the area of compound figures. This topic is often included after students have learned how to calculate the area of rectangles and squares, but before they explore the area of parallelograms, triangles, and other shapes. So, is this topic simply "applications" where students can use their newly learned knowledge of calculating the area of rectangles and squares?

There are multiple ways to find the area of compound figures. One can sub-divide the given shape into a collection of shapes for which one already knows how to calculate the area. Or, one can dissect the given figure and re-arrange the pieces to create a familiar shape. The first method draws on the idea that the area of a figure is equal to the sum of the areas of its parts. The second method draws on the related idea that cutting and rearranging pieces of a figure does not change the area. These same ideas can and should be the foundation for approaching the area of parallelograms, triangles, and other shapes. By making sure students understand these ideas, the topic of area of compound figures can serve as a bridge to the future topics of areas of other shapes.

In general, then, planning a lesson necessarily involves consideration of how the ideas in it connect to previous and future ideas.

Providing meaningful tasks or problems

The problem or task you will provide in the lesson must help students develop the goal understanding. It should also capture students' attention. Thus, setting up the problem/task in the context that is meaningful to students is very important. The point is not necessarily to give students "real-world" problems. Rather, we want capture students' attention so that they generate their own questions. It is these students' own questions that will drive the lesson. Therefore, teachers must select problems/tasks carefully so that students' own questions are indeed productive ones for moving them closer to the learning goals. Paying attention to students' sense of curiosity and surprise may be helpful.

Anticipating and planning for students' difficulties

When students are learning mathematics, they encounter many challenges. Identifying possible challenges is an important part of good lesson planning. Research has identified many common errors and misconceptions students make. Besides those common errors and misconceptions, some students may have difficulty understanding a particular problem/task. The more possible challenges you can identify, the better prepared you can be to handle them.

Based on the challenges you identified, you can think about how the problem or task may be posed to the students. The purpose is not always to avoid the challenges. It may be beneficial for students to explicitly discuss particular challenges in a lesson. However, having multiple contingency plans will allow teachers to deal with those challenges more effectively as they implement the lesson.

Anticipating students' responses that include misunderstandings/misconceptions.

Although the problem/task selected for the lesson leads students to new understanding, it must be also accessible to all students with their current understanding. That means the problem/task should have multiple entry points and multiple solution paths. It is essential that teachers actually solve the problem/task they are considering for the lesson themselves - perhaps trying to solve it in as many different ways as possible, using only what students currently understand.

Anticipating how your students will solve the given problem/task is important not only for ensuring that the problem/task is accessible to all students but also to help teachers develop a plan (or a set of multiple plans) to orchestrate the classroom discourse to guide the class discussion for level-raising. Japanese teachers call this process *neriage* (need a link to the glossary). By carefully sequencing how students' ideas are shared and discussed, teachers can orchestrate the classroom discourse to guide students to the goal understanding.

Using learning aids for all the students to access the tasks. Access to necessary materials and technology

To facilitate students' problem solving activities, various learning aids may be useful. Those aids include, but are certainly not limited to, manipulatives, measuring instruments, graph paper, calculators, and computers. In deciding which aids to provide, it is important to consider how they would influence student thinking, against the goals of the lesson. For example, you may not want to provide grid papers while working with an L-shape area problem if the goal is to have students focus on ways to calculate the area, since the grid may invite students to fall back on counting squares. On the other hand, some students may need to physically cut the given figure with scissors to understand that the shape may be sub-divided or re-arranged.

In addition to the question of what materials to provide, there is the question of how those materials will be made available. If the teacher provides concrete manipulatives up front, some students who might otherwise have been ready for more abstract thinking may instead rely on the manipulatives. An alternative is to make students request the materials when they need them.

Sometimes the quantity of materials available will influence how students must be grouped. Another consideration is how those materials may be used as students share their ideas. In some cases larger copies of the materials may have to be prepared to facilitate public discussion.

Employing teacher questioning and supports for student learning – Key questions to enhance students' learning

When we teach through problem solving, students' own reasoning and questioning play the central role in a lesson. However, because students are still developing their understanding, sometimes they may not be able to ask the key questions that might move them to the next level. Thus, teachers may have to ask those questions on behalf of the students, for now. Therefore, teachers must think about key questions that may initiate, advance, and deepen students' thinking processes. Some questions must be asked to help students reflect, evaluate and summarize their learning in the lesson. Questions that ask students to recall and connect their prior experiences to the current problem/task may be useful to help students initiate their thinking processes. Questions that ask students to compare and contrast various ideas might be useful to deepen students' thinking. In all cases, teachers have to carefully think about how—and when—questions will be posed.

Grouping student to provide opportunities for them to express their ideas

How students will be organized at different points in a lesson should be carefully considered by teachers. As teachers think about the organization of their students, they must carefully balance the need of individual students to think and solve problems independently and their need to exchange ideas with their peers. While students are working individually, they will not be exchanging ideas with anyone else, except with the teacher if he or she comes around. During group work, students will have opportunities to freely exchange ideas, but the teacher may not know what took place. During whole class discussion, teachers can orchestrate the exchange of ideas, but only a relatively few students may have the opportunity to express their thinking.

Each mode of organization can promote students' learning if used appropriately. On the other hand, students' learning can be hindered by inappropriate grouping of students. Therefore, the choice of students grouping should be made intentionally by teachers. If external circumstances impose a particular mode of organization, teachers should develop contingency plans to deal with any possible difficulties that might arise due to the organization.

Identifying additional tasks/problems to extend/secure the students learning.

In many lessons, it is helpful and necessary to have additional tasks to help students solidify and extend their new understanding. If a new procedure is developed during a lesson, a set of practice problems may be appropriate. In any case, the additional tasks should be closely tied to the learning objective of the lesson, and how they should differ from the initial task must be carefully considered, since they are still a part of the mathematics story line of the lesson.

Measuring students learning

As assessment becomes an integral part of instruction, teachers must think about how to assess their students' understanding throughout the lesson. So, as teachers carefully consider what the learning objective of a lesson is going to be, they also consider how they would know if students have developed the target understanding. Teachers may want to carefully articulate what it means to have the goal understanding. How will the students' problem solving be different if they do or do not yet have the goal understanding? Teachers may also think about when to ask those questions to monitor and advance students' thinking processes. The results from an assessment question may play a role in how students may be organized during the lesson.

Clearly, the results of an assessment question (or a set of assessment questions) toward the end of the lesson will influence not only the planning for the next lesson but also how the lesson will be analyzed and revised. The following two questions may provide a useful framework to think about the assessment questions:

What specific mathematics does this question involve? The assessment question should be about the specific mathematics the lesson focused on.

How does the question get at students' understanding of the specific mathematics? How will the response by those with the desired understanding be different from those who have yet to develop the goal understanding?

Key Considerations in Analyzing and Revising Lessons

As we analyze and revise lessons, our efforts should be guided through the specific factors that guided lesson planning and implementing discussed in Section III. Listed below are some questions that might frame your analysis and revision of mathematics lessons. Although questions are often stated in a yes/no format, it is very important that we discuss evidences from the lessons to support our answers. That is, whether we answer any question affirmatively or negatively, we should be able to share specific instances from the lesson that will support our response. Some follow up questions are suggested to deepen and guide your analysis and revision. Often times, answers may be affirmative with some students but not with others. Being able to contrast those students may help us evaluate various hypotheses we may have about the factors contributing to students' success (or lack thereof).

Identified clear objectives that enhance students' learning

- Did the students see the connection between the topic that students previously learned and the one that they learned during the class?
- What seems to prompt students to make this connection?
- What changes might promote students to make connections more effectively?
- Did the lesson activities encourage students to learn contentiously after the lesson?
- Did students understand how and why the topic is important?
- Did students accomplish the learning objective(s)?

Provided tasks and learning aids that help students accomplish the learning objectives

- Was the degree of challenge appropriate for the students at the time?
- If not, what seems to be missing?
- Was there any unanticipated response?
- Did the lesson provide opportunities for students to express mathematical ideas and thinking process in individual writing and the class/group discussion?
- What change might promote more effective exchange of ideas?
- Did the lesson incorporate appropriate use of visualization and communication tools that include board writing?
- Did the lesson provide students to extend/secure their knowledge/understanding/skills (extension problems and/or exercises during the lesson and outside the class)?

Employed teacher questioning and supports for students learning

- Did teachers’ questions and guidance enhance students learning?
- Which questions seem to enhance students' learning?
- Did teacher provide appropriate support for students to overcome misconceptions and misunderstanding?
- What was the misconception or misunderstanding and what teacher support promoted students overcome it?
- Did teacher provide students to access to necessary materials and technology?
- Was grouping (individuals, pairs, small groups, whole class) used appropriately to maximize students learning?

Effective integration of assessment

- Did the teacher use formative assessments to make decisions to modify/adjust the plan to maximize students learning?
- Were the methods for evaluation appropriate?

Revise and present lesson with improvements reflecting lesson analysis

As teachers revise the lesson, they may want to review the factors discussed in Section III and consider how they can be more refined. For example, in some cases, the learning objective(s) may have to be further specified. In other cases, teachers may want to consider the type of learning tools used in the lesson, or how they were presented to students. Perhaps assessment questions may have to be revised. In any event, the revision process should be informed and guided by the analysis of the data collected during the lesson.

Examples of Using the Guide to Analyze Lessons

This section presents analyses of several videotaped lessons on this web site as examples of how this guide might be used.

Guide for using "Do I Have a Window Seat or an Aisle Seat?"

In this section, we will review a 5th grade lesson from Japan titled, "Do I Have a Window Seat or an Aisle Seat?", and analyze it using the guidelines.

Learning Objectives

According to the instruction plan, the goals of the lesson are (p. 5),

- Students can think about how to categorize whole numbers from their own point of view.
- Students can understand the merit of paying attention to the reminder that results when a whole number is divided by a certain number.

During the summary section of the lesson, students were given two additional numbers to check their locations. From the video, we can tell that the majority of students used division to determine the seat locations. In addition, when teacher asked why they used the division strategy, a number of students indicated that the strategy was easy and quick. Thus, this lesson seems to have achieved its learning objectives.

Learning Tasks

In addition to these mathematical goals, the research team is working on helping students become autonomous problem solvers (p. 1). In particular, they are exploring how to draw out students' own questions that can guide their learning more effectively. The planning team felt an interesting (to students) learning task is important. Moreover, the team decided to pose a question with insufficient information so that students will ask for additional information.

Watching the introductory stage of the lesson, it is clear that students were genuinely interested in determining whether or not their seats are by windows. Furthermore, the initial question posed with insufficient amount of information prompted students to request a number of additional information. These ideas contributed to students making the learning task their own task.

Teacher Support

Although students understood what the problem is asking, there were some students who were struggling to get started. The teacher called those students to the front of the classroom to provide additional support. He first told those students that at any point if they knew what they needed to do, they could go back to their seats. The teacher asked students to determine a particular seat number by pointing to the seating chart. Students were able to tell the seat numbers immediately. So, the teacher asked how they knew and made a few suggestions what they could possibly do to further extend their ideas to solve the problem. As we can see from the video, this brief session seemed to provide just right amount of support, not too little nor too much.

During the remainder of the independent problem solving time, the teacher circulates around the classroom. He asks some clarifying questions and makes some suggestions to extend students' thinking. He also records the methods students are using in the seating chart.

Incorporating Assessment

During the whole class discussion, the teacher calls on Ichikawa-kun first, who recognized that the numbers were increasing by four. The teacher then had him arrange the number cards (1, 5, 9, 13 ...) on the board. The next child noted that the number cards represented the window seats on the left side of the train, and the right side window seats will be one less than these numbers. The number cards for the right window seats (4, 8, 12, 16 ...) were posted.

Yoshikawa-kun then used the four's multiplication facts. Since his number was 47, he noticed that it was 1 seat away from a multiple of 4, 48, therefore, his seat was not a window seat.

Then the boy in red shirt proposed the idea of using division. His number was 55, so when he divided it by 4, he obtained the quotient of 13 and the remainder of 3. He noted that the seat numbers that are divisible by 4 or those with the remainder of 1 will be window seats.

The discussion followed very closely to the lesson plan. This was made possible by the teacher carefully noting which students were using which strategies. Thus, even though he might appeared to be calling on volunteers to share their ideas, he was carefully orchestrating the whole class discussion, using the data he collected as he circulated among students during the independent problem solving time.

As discussed earlier, during the summary segment of the lesson, the teacher asked students to determine whether or not two additional seat numbers were window seats. The video shows that most students were indeed using the division method. Not only were students using the division method, they were doing so intentionally because they felt that was the most efficient method.

Revising the lesson

There are a couple of points from this lesson that may be worth further reflections. First, the students were given an empty seating chart they could use. Although the teacher told the class that they did not have to use it, the availability of the chart may encourage more students to use the chart. The question is whether or not that will actually limit students from using more mathematically sophisticated method. If students' focus is solely on answering the question, that might be possible. On the other hand, if students are interested in finding out patterns that might be useful in answering the question, having actual seat numbers may be helpful.

Another point to consider is that the orientations of the seating chart as presented originally (and also on the chart handed out to the students) and the way the number cards were posted on the blackboard were different. When students began discussing right/left window seats, translating from one chart to the other might have been confusing to some children.

Guide for using "Thinking Systematically"

In this section, we will review a 6th grade lesson from Japan entitled, "Thinking Systematically", and analyze it using the guidelines.

Posing the problem

The beginning of this lesson, (link to the video) is designed to engage students' interest. The teacher orients them to the task by asking students to guess what the lesson might be about. He relates mathematics to life outside school by asking questions such as how much students would pay for pens and pencils. This builds the "productive disposition" strand of mathematical proficiency. The segment ends when the teacher poses the problem for the lesson.

To mathematics experts, this problem is a standard algebra problem, with two unknowns and two linear equations. However, these students are too young for algebra and the intention of the lesson is to give them experience of tackling problems systematically. The lesson is not about algebra. The problem has been deliberately chosen to highlight thinking strategies. Every aspect of the problem has been considered in the planning, including the size of the numbers, their numerical relationships, as well as the mathematical structure. In the first part of the lesson, the students focus on the problem to gain experience of the benefits of using a table. This experience is discussed at the end of the lesson.

PROBLEM: You bought some 40 yen pencils and some 70 yen ball-point pens. There were 10 of them in total for 460 yen. How many pencils and ball-point pens did you buy?

Understanding the problem

In the second section, (link to the video) the teacher begins by ensuring that all students understand the problem. He does this by first asking students to read the problem aloud, and then by asking the class some simple questions related to the problem. In this way, the lesson models Polya's first phase of problem solving: understand the problem. The teacher then asks students to suggest how the problem can be solved, and he writes important suggestions ("multiples", "tables") on the board to emphasize that these are important.

The construction of the table is very carefully managed by the teacher, who stresses the importance of using a labeled table, and shows very strong evidence of lesson planning.

A student suggests labeling the rows of the table “pencil” and “ball-point pen”. The teacher implicitly corrects this by bringing out the pre-prepared label “number of pencils” and “number of ball-point pens”. This is an important distinction to make when learning algebra later, because students who believe a letter stands for an object (such as a pencil) rather than a number (such as the number of pencils) make systematic errors when setting up equations.

The teacher has prepared strips of paper with three spaces that hold one number of pencils, the corresponding number of ball-point pens and the total price. So, for example, on one strip of paper the teacher can write 1 (pencil), 9 (pens), 670 (yen). Having 12 strips that hold the 3 quantities, rather than 3 horizontal strips (e.g. one for pencils, one for pens, and one for total) is a subtle point of the lesson design. This emphasizes the vertical relationships between the quantities that are the key to solving the problem (e.g. number of pencils = 10 – number of pens). For this problem, these “vertical relationships” are more important than the “horizontal” relationships. It is also interesting that, at this early stage of the lesson, a student in the class notes that the total price for 1 pencil and 9 pens is 30 yen less than the total price for 0 pencils and 10 pens. This difference of 30 will be the key to solving the problem. However, at this stage the teacher is not ready to incorporate this comment into the flow of the lesson. Perhaps he wants all students to find this for themselves, rather than taking it from one student.

In this lesson, the teacher calls upon many students, and conducts whole class discussion. In this way, he is able to monitor the degree to which the various class members are following the lesson. The extent of engagement of the whole class cannot be judged from the video, but we see the teacher taking steps to monitor this for himself at all points of the lesson.

Individual Work and Class Discussion

In the third section, (link to the video) students construct the tables in their notebooks and some students write their combinations and total price on the strips placed randomly around the board. In the fourth section, (link to the video) the calculations are checked, the strips are checked for completeness and the answer is identified.

The teacher then draws the class’s attention to the “confusion” with strips randomly arranged, and two students come to the board to make a systematic table using the pre-prepared labels. The class then discusses why arranging the entries systematically is good. They identify the patterns that are evident and note again the fact that when the number of pencils increases by 1, the total price decreases by 30 yen. The table makes this pattern easy to see. Students in the class suggest the reason for this and the teacher gives two students the opportunity to try to express this relatively complex mathematical pattern clearly in words. Looking at the table, students observe several patterns (e.g. the total price decreases by 30 yen moving from the left and increases by 30 yen moving from the right).

During the class discussion, a student unexpectedly comes to the board and draws a graph showing the number of pens as a function of the number of pencils (i.e. part of the line $y = 10 - x$). The teacher illustrates what he has done to the class by joining points on a histogram. This very clear explanation is targeted directly to students at Grade 6 level and it give evidence of the teacher being very well prepared mathematically, so that he can help the many students to benefit from the insight of one.

Further Discussion (HaKaSe)

In the fifth section, (link to the video) the teacher reminds the class of the HA-KA-SE (fast-easy-accurate) principle that they know well, and they discuss whether the table method is indeed fast, easy and accurate. The children decide it is not fast, and would be too slow if the number of pens and pencils was large. This leads for a search for a fast method, and students suggest possibilities, although they all struggle to verbalize these.

Note how the students and teacher write the unclosed expressions on the board. For example, they write (70 – 40) rather than 30. This is a characteristic of Japanese mathematics education which emphasizes relationships.

The suggestions from the class at this point of the lesson have to be handled carefully. Students make suggestions and improve upon each other's suggestions. For example, one student explains: "What he wanted to say was 460 – 40 x 10 = 60 and 60 ÷ (70 – 40) = 2 and that (2) is the number of pens bought" Another student explained the meaning of the first expression as "If you buy 10 pencils, the change is 60 yen. Divide by the difference of prices, which is (70- 40) yen". Later, the teacher summarizes these contributions and explains the logic behind the method of solution by using the table extensively. This part of the lesson very clearly illustrates the development of two aspects of mathematical proficiency: strategic competence and adaptive reasoning, as well as the teachers' detailed preparation and thinking ahead as to how this can be explained well.

Another student suggests starting at the other end of the table, purchasing 10 pens for 700 yen but with time running short this is not followed up. Wishing the students to practice using their new idea, the teacher then gives students a new problem with the same mathematical structure to solve individually.

Closing the Lesson

At the end of the lesson, (link to the video) the students are asked to suggest the title of the lesson. In their reflection, they say it will involve tables and mathematical expressions. The teacher ends the lesson by naming it "From tables to mathematical sentences".

The lesson has shown a strong 'story line". It started with a problem that is strong enough to carry the whole lesson. There was a clear period of deepening understanding; the follow up problem closely followed the main idea developed by the students, and the lesson ended with a time where students reflected on their mathematical learning.

References

Greer, B. (1992). Multiplication and division as models of situations. In (Grouws, D. A., Ed.) Handbook of research on mathematics teaching and learning, pp. 276–95. New York: Macmillan Publishing Co.

Hiebert, J., and Grouws, D.A. (2007). The effects of classroom mathematics teaching on student's learning. In F. K.Lester (Ed.), Second Handbook of Research on Mathematics Teaching and Learning. Greenwich, CT: Information Age.

Leinwand, S. (2009) Accessible Math. Portsmouth, New Hampshire: Heinemann

National Council of Teachers of Mathematics (2000). Principles and standards for school mathematics. Reston, VA: The Council.

National Council of Teachers of Mathematics. (2006) Curriculum Focal Points. Reston, VA: The Council.

National Research Council. (1989). Everybody Counts. Washington, DC: National Academy Press.

National Research Council (1999). Global Perspectives for Local Action: Using TIMSS to Improve U.S. Mathematics and Science Education. National Academies Press.

National Research Council (2001). Adding it up: Helping children learn mathematics. Washington, DC: National Academies Press.

Polya, G. (1945). How to solve it: A new aspect of mathematical method. Princeton, New Jersey: Princeton University Press.

Sugiyama, Y. (2008). Introduction to elementary mathematics education. Tokyo: Toyokan Publishing Co.

U.S. Department of Education. (2008). Foundations for Success: The Final Report of the National Mathematics Advisory Panel. Washington, DC.